Technical Note

Measurement Uncertainty for PHOM Test of Germanium

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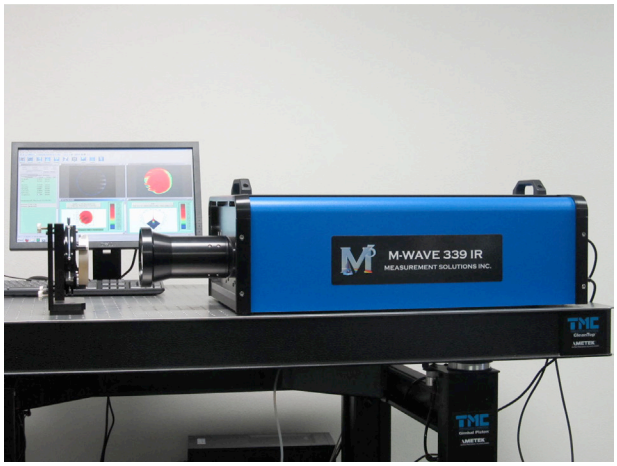

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Abstract

A standard measurement technique for optical material index homogeneity is the PHOM or polished homogeneity test. Previous papers have reported PHOM test reproducibility. This paper explores PHOM measurement uncertainty and then estimates the measurement uncertainty for a 3.39 um wavelength measurement Ge material. It is determined that several parameters primarily drive the "Interferometer" and "Practice" measurement uncertainty. These parameters are Ray Trace, Peak Pixel Deviation, Reproducibility, and Air-Turbulence and Thermal Gradients. The combined uncertainty (Uc) for the Interferometer and Practice is estimated at ± 4.8 ppm 2**σ** for a 17 mm thick 100 mm diameter part in a 0.1°K stable environment.

The biggest contributor to measurement uncertainty is the optical property dn/dT of the germanium itself. Germanium dn/dT = 396 x 10^{-6} /°K indicates that thermal equilibrium within the part be <0.001 °K. This level of temperature control demands careful attention to all sources of heat, including the laser itself.

Introduction

Optical material homogeneity is an important contributor to optical system performance. Measurement of homogeneity has evolved into a standard interferometer based test. The polished homogeneity test or PHOM test has been in use for over 30 years.[2] Further PHOM was shown to have advantages over oil-on-flat homogeneity testing[3], yet the authors are not aware of a published estimate of measurement uncertainty (Uc) for PHOM. This paper establishes measurement uncertainty from first principles, enabling users to estimate Uc for a particular PHOM test once the parameters are measured or estimated.

PHOM Test

The PHOM test nominally reports only part homogeneity. The test is structured to minimize errors from the test part and measurement cavity surfaces leaving only the variation in homogeneity. Four measurements are made during the test: The transmitted wavefront of the test part within the cavity , test part Surface 1 to the transmission flat, test part Surface 2 to the transmission flat and finally the empty cavity (Figure 1a-1d below). From these four measurements homogeneity is isolated. In Figures 1 the optical paths are exaggerated for use later analyzing measurement uncertainty.

Test Part Specifications[4]

Surface Finish	Polished, 300 nm PV
Aspect Ratio (Thickness to Diameter)	≤ 6:1
Part Wedge by Size	100 mm: 5 to 25 arc minutes 150 mm: 3.5 to 35 arc minutes 300 mm: 2 to 66 arc minutes 400 mm: 1 to 85 arc minutes

The test part is wedged enabling the measurement of one surface at a time.

Measurement of Homogeneity

Each of the four measurements (M1 through M4) contains measurement errors that are minimized while calculating homogeneity in equation 1 below. The interferometer analyzed is Twyman-Green

[1] Contact person: rsmythe@apre-inst.com

[2] J. Schwider, R. Burow, K.E. Elssner, R. Splaczyk, & J. Grzanna; "Homogeneity testing by phase sampling interferometry"; 15 September 1985 / Vol. 24, No. 18 / APPLIED OPTICS

[3] A. Fanning, J. Ellison & D. Green, "Polished Homogeneity Testing of Corning Fused Silica Boules", SPIE Conference on Optical Manufacturing and Testing III, Denver, Colorado • July 1999, SPIE Vol. 3782

[4] Polished Homogeneity MetroPro Application Note©, Zygo Corporation OMP - 0386B, 11/96

configuration; for simplicity a Fizeau configuration is drawn. In a Twyman-Green the Transmission flat is replaced by a Beamsplitter and Reference mirror combination.

Empty Cavity

$$M1 = R(x, y) - T(x, y)$$

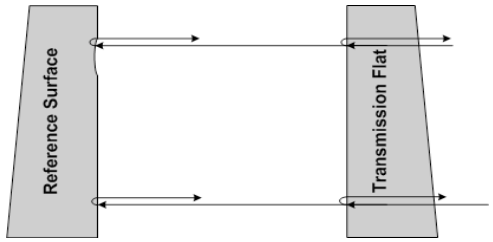

Figure 1a

Entrance surface S1 as a reference

$$M2 = S1(x, y) - T(x, y)$$

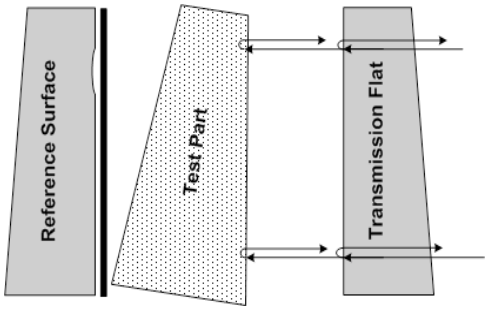

Figure 1b

Exit surface S2 as a reference

$$M3 = S2(x, y)\cdot n + t_0 \cdot \overline{\Delta n}(x, y) - S1(x, y)(n-1) - T(x, y)$$

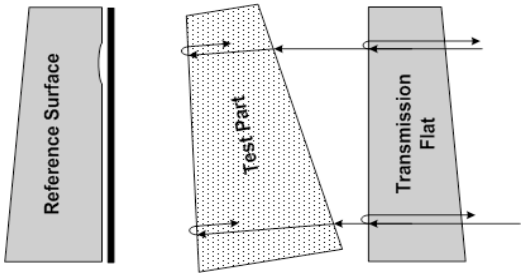

Figure 1c

Part in Transmission

$$M4 = R(x, y) + S2(x, y)(n-1) + t_0 \cdot \overline{\Delta n}(x, y) - S1(x, y)(n-1) - T(x, y)$$

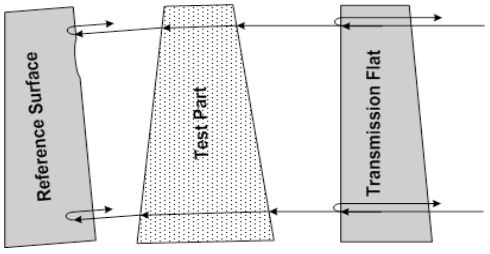

Figure 1d

$\overline{n}(x, y)$ ≡ Part Index Homogeneity, the target measurement.

t_0 ≡ Part Thickness (this varies across the part see Appendix 1 for explanation)

n ≡ index of refraction for the test part

$$\overline{n}(x, y) = \frac{n(M4 - M1) - (n-1)(M3 - M2)}{t} \qquad (1)$$

For more details see Appendix 1.

The beauty of this test is primary error sources (cavity errors, and part surface errors associated with topography of the surfaces involved) are minimized, and only additional error sources require consideration.

Definitions

Measurand

Measurand is the "Particular quantity subject to measurement."[5] The raw data measurand for PHOM is an array of points reporting Δn_{par} at each camera sensor pixel.

Filtering, via the instrument transfer function or mathematical application contributes to the definition of measurand and ultimately the Uc. If a smoothing function is applied or low pass or high pass filter, these too must be considered as part of the measurand.

Reported results define the PHOM measurand of Δn. Homogeneity RMS, the root-mean-square deviation of all individual Δn points from a plane fit and PV, the difference between the highest and lowest Δn points from a plane fit. For this paper only RMS will be discussed as PV is an unstable result driven by two points, hence noise prone.

The measurand for this paper will be the RMS of the raw Δn data, without filtering.

Uncertainty of Measurement

Uncertainty of measurement (Uc) is defined as, "parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand."[6] Therefore Uc cannot be known, only estimated from known error sources and best guesses for considered sources of error that are not quantifiable. Uc is the reasonable spread of results that can be expected from a given measurement. If two measurements give different results, with overlapping Uc's then the measurements agree within the Uc.

Any PHOM test will report a result, yet "any measurement that you make …without knowledge of the uncertainty… is meaningless", Professor Walter Lewin.[7] Knowing the uncertainty minimizes the arguments regarding what measurement is "right". Further if two measurements disagree and do not have overlapping Uc's then the knowledge of the uncertainty in one or both measurements is underestimated.

Uc is typically underestimated because we do not know what we do not know. Early Hubble Constant measurements were reported to have a Uc of 10%, we now know they had a 1000% error in Uc estimation. It took 75 years to narrow the actual Hubble Constant Uc to 3%.[8]

Uncertainty Budget

Uc is driven by several factors: Calibration, people, environment, standards, equipment and procedures. The standard method to estimate Uc is an Uncertainty Analysis/Budget. ISO/TS 14253-2:1999 lists sources of uncertainty:

Source	Factors to Consider in PHOM Test
Environment	Absolute Temperature effects values of n and thickness Temperature Gradients effect: test part birefringence and index, instrument, reference, reference wavefront, return flat and test surfaces. Vibration causing repeatability and reproducibility issues.

[5] "International Vocabulary of Basic Terms in Metrology", Section 2.6, ISO, 1993

[6] "International Vocabulary of Basic Terms in Metrology", Section 3.9, ISO, 1993

[7] We want to thank Dr. Hy Tran of Sandia Labs for an introduction to this video link and his course on Measurement Uncertainty. https://www.youtube.com/watch?v=PmJV8CHIqFc, listen between time = 4:40 and 5:10 (mins:secs)

[8] M. Livio & A.G. Riess, "Measuring the Hubble constant", Physics Today 66(10), 41 (2013); doi: 10.1063/PT.3.2148

	Air turbulence and static thermal gradients within measurement cavity.
Reference Element	Interferometer reference surface and in the Twyman-Green configuration the reference/beamsplitter influenced reference wavefront
Measurement Equipment (Interferometer)	Sensor pixel count, wavefront collimation, wavefront error, ray tracing errors due to non-nulled cavity, repeatability and reproducibility of measurement, pixel noise and coherent pixel-wise noise.
Measurement Set Up	Mechanical distortions of optics and test part, stress and birefringence induced by mechanical mounting; time waited to thermally “soak” the part (adequate?). Test part shift between measurements
Software & Calculations	Rounding, calculation, and digitization errors
Metrologist	Hot or cold hands, careful setup and rigorous following of procedures and systematic bias’ in set up
Test Part	Flatness, intrinsic birefringence wedge and wedge uncertainty, amount of homogeneity, index of refraction, coefficients of thermal expansion & $\Delta n/\Delta T$, lateral shift of the part when tilting causing differential errors.
Measurand Definition	Noted above
Measuring procedure	Does the measurement vary if different procedures are followed.
Physical Constants	Wavelength of light

Type A and B Evaluation

Two measurement uncertainties are defined: Type A and type B. Type A are measurable, like repeatability. Type B are estimated from good scientific judgement (guesses). Both are treated equally. Type A and B designations are different from past thinking considering systematic and random error sources, there are only type A and type B.

Sensitivity Coefficients

Sensitivity coefficients translate known quantities, like temperature, into the quantity of interest like Δn. Known thermal sensitivity of the measured material index of refraction, $\Delta n/\Delta T$, translates knowledge of thermal uncertainty into measurand uncertainties. For temperature this is a direct calculation. For ray tracing error estimation is difficult as interferometer designs have varying sensitivity, and the measuring procedure can vary and they directly affect Δn results.

Analyzing the Uncertainty Budget

A Root Sum Squared (RSS) method is used to combine the results. The variable above need to be selected as best possible for non-correlation and also the worst case result of all variables trending the same direction has a statistically low probability. Plus “the central limit theorem states that the distribution of the sum (or average) of a large number of independent, identically distributed variables will be approximately normal, regardless of the underlying distribution.”[9] Therefore the Uc budget is treated as a sum of uncorrelated normal distributions in a RSS fashion.

[9] http://www.math.uah.edu/stat/sample/CLT.html

Expanded Uc

The RSS reported above provides a one-standard deviation (1**σ**) estimate of Uc. Typically this is expanded to a 2**σ** estimate by multiplying by a factor of 2, for a 95% confidence in the Uc reported. The Uncertainty = 2*Uc and needs to be reported.

Evaluating Error Sources

Test Set Up

Interferometer: 100 mm aperture Laser Twyman-Green 3.39 um wavelength interferometer.

Properties and Environmental Assumptions

- Material Ge
- Diameter = 100 mm, Thickness (d) = 17 mm (OPD assumed to be 8.5 um [5 ppm])
- Thermal Expansion Coefficient (CTE): $5.9 \cdot 10^{-6}$ / °C
- dn/dT = 396 x 10^{-6} /°C[10]
- Room temperature: Range of T = ±0.5 °C
- Thermal Conductivity: 60.2 $W \cdot m^{-1} \cdot K^{-1}$ (Al = 167 $W \cdot m^{-1} \cdot K^{-1}$ and Glass = 1.1 $W \cdot m^{-1} \cdot K^{-1}$) Germanium will achieve thermal equilibrium 60X faster than optical glass and 2.5 X slower than AL.
- Assumed ΔT = **0.1° K** of thermal equilibrium.

Test: The PHOM test will match the drawings above.

Practice: Good practice will be assumed. The part will achieve thermal equilibrium, system is vibration isolated, nominal air turbulence is controlled with minimal air path in the set up.

Interferometer Uc Contributions

Type	Source	Estimated Error	± Uc Contribution
A	Laser Wavelength	1 ppm	<1 nm[11]
A	Reference Wavefront	30 nm	~5 nm[12]
B	Collimation[13]	power growth over 2*n*thickness	<1 nm
B	Ray Trace Errors	10 nm/fringe	10 nm[14]
B	Peak Pixel Deviation[15]	~5 nm	20 nm

[10] A. Mann, “Infrared Optics and Zoom Lenses, Second Edition (SPIE Tutorial Text Vol. TT83)”, Chapter 3, p 27, SPIE Publication

[11] Wavelength errors affect the length measurement. The part has a 68 mm OPL, with 10 PPM variations the error is <1 nm

[12] This error is a first order error mostly removed by the test procedure, the higher spatial frequency errors in the wavefront will shear causing differential errors with larger **relative** errors occurring in the high frequency errors.

[13] Collimation must be considered separately for a Laser Fizeau, here the reference and collimation are treated as one.

[14] One-half fringe tilt nulling assumed for four measurements then RSS’d for error estimate.

[15] Peak Pixel Deviation is defined by the 99.5th percentile of the pixel-wise standard deviation map for 36 sequential measurements (4 averages); this result measures time varying behavior per pixel - relevant to PHOM high spatial frequency measurements - Scaled from Zygo specification sheet.

Practice Uc Contributions

Type	Source	Estimated Error	± Uc Contribution
A	Reproducibility	16 nm	16 nm[16]
B	Refraction through part 5 arc minutes wedge	shift of 4 mm/meter[17] See discussion below	3 nm[18]
A	Air turbulence and static thermal gradients in Fizeau cavity	7 nm	7 nm[19]

Reproducibility and air-turbulence above cover the environmental factors in the Uc budget.

Refraction Through Part Error

The data here was extracted from a 2002 paper characterizing PSD of several optical materials.[20] Ge was not characterized, yet all materials demonstrated similar behavior. The data runs from 1 mm to 1 nm spatial WAVELENGTH, yet the curve demonstrates the well known ~2.5 log (PSD_{2dD}) for each order of magnitude spatial frequency change.

With a 1K X 1K camera with a limited imaging system to two-orders of magnitude resolution, the finest details imaged are 1 mm across.

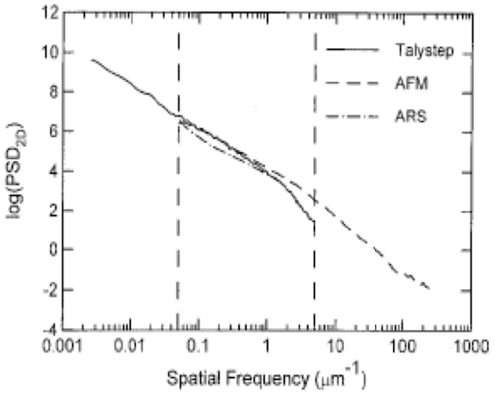

Fig. 23. PSDs for polished fused-silica SQ1.

The Ge wedged part of 100 mm diameter with a 5 arc minute wedge induces a 1 mm shift if the spacing between the reference surface and test part is 250 mm. The expected 1 mm shift noted above due to part refraction is just resolved and no spatial filtering is expected. The errors induced by this shift are as large as the expected height errors over 1 mm as predicted by the PSD curve.

Integrating the PSD curve over the full aperture (100 mm) and then over 1 mm yields a relative surface height amplitude error over the 1 mm shift. The RMS ratio for the RMS (100 mm to 1 mm) is 237/5.6 indicating the induced error for the 30 nm surface is <1 nm RMS or **3 nm PV.**

[16] Results cited by A. Fanning, J. Ellison & D. Green, "Polished Homogeneity Testing of Corning Fused Silica Boules" used.

[17] Measuring transmitted wavefront to reference surface (figure 1d) the beam shifts across the reference surface compared to the cavity measurement.

[18] Assume a 500 mm cavity the lateral shift at the reference, assuming a centralized test part = 1 mm or 4 mm * 0.25. This shift error depends on the spatial frequency content of the reference surface, discussed in text below.

[19]This can be only achieved for a very short cavity lengths - the overall length of the cavity should be as short as possible. Covering the Fiezau cavity with a rigid enclosure could be considered as an option but it should not be treated as an alternative to short cavity length because of increase probability of static thermal gradients under the enclosure.

[20] A Duparre´ , J Ferre-Borrull, S Gliech, G Notni, J Steinert, and JM. Bennett, "Surface characterization techniques for determining the root-mean-square roughness and power spectral densities of optical components", p154 APPLIED OPTICS Vol. 41, No. 1 1 January 2002

Part Uc Contributions

Type	Source	Estimated Error	± Uc Contribution
B	Thickness(dt/dT)	$dt_{part} = \Delta T * CTE * d$ = 1 E-8 meter	<<1 nm[21]
B	Thickness(dn/dT)	$dn = \Delta T * 4 * 10^{-4}$ $= 4 * 10^{-5}$	0.6 nm[22]
B	OPL (dn/dT)	$= 4 * 10^{-5} * 0.17$ m	700 nm
B	Thermal Stress Birefringence	Discussed in the analysis section below	Set up dependent
B	Mechanical Stress Birefringence		
B	Inherent Birefringence		1 nm to 1,000 nm

Total Measurement Uncertainty (Uc)

Instrument Uc

Total Uc for the Instrument is the RSS of all instrument related error sources:

Laser Wavelength, Ref. Wavefront, Collimation, Ray Trace, Peak Pixel Deviation

RSS = $[(1\text{ nm})^2 + (5\text{ nm})^2 + (1\text{ nm})^2 + (10\text{ nm})^2 + (20\text{ nm})^2]^{½}$ **= ±23 nm Uc 1σ**

On a 17 mm thick part this is ±2.6 ppm 2σ

Practice Uc

Reproducibility, Refraction, Air Turbulence

RSS = $[(16\text{ nm})^2 + (3\text{ nm})^2 + (7\text{ nm})^2]^{½}$ **= ± 18 nm Uc 1σ**

On a 17 mm thick part this is ± 2.2 ppm 2σ

Part Uc

Thickness (dt/dT, Thickness (dn/dT), OPL (dn/dT)

RSS = $[(1\text{ nm})^2 + (0.6\text{ nm})^2 + (700\text{ nm})^2]^{½}$ **= ± 700 nm Uc 1σ**

On a 17 mm thick part this is ± 80 ppm 2σ

By far the largest contributor to Uc is the part parameter dn/dT, which is an order of magnitude larger than any instrumentation or practice uncertainties.

[21] The contribution follows from the first derivative of equation 1, $d\Delta n\,(T) = -\,dt_{part}(T) * n*dt_{part}/t_{part}^2$

[22] The contribution follows the first derivative dn(T) equation 1, $d\Delta n\,(T) = dn(T) * t_{part}$

Analysis

Improving the Instrument and Practice Results

For now ignoring Part Uc, the PHOM test under these environmental parameters can achieve a SCHOTT homogeneity level test of Grade H2 (±5 ppm 2σ).[23] To improve Uc to a Grade H3 (±2 ppm 2σ) the dominant errors of ray tracing, peak pixel deviation, reproducibility, air-turbulence and thermal gradients must be reduced. The following are strategies to follow:

Ray Trace Error: The best method to minimize these errors to is to null the cavity before each measurement. The nulling must be assisted by reporting X and Y tilt in the cavity. Decreasing tilt to <60 nm across the aperture will minimize these contributions. With the 3.39 um wavelength this is only 1/50 wave, tilt difficult to see visually.

Peak Pixel Deviation: Minimizing PPD primarily entails reducing coherent noise in the data and pixel noise in the camera. The coherent noise arises from back reflections off optics, glints, scratches and dust. Also reflections off internal surfaces that find their way into the camera. Camera noise contributes directly and is mostly out the control of the user. This is a difficult parameter to control and must be measured to confirm its contribution, since we assigned it a B type variable (educated estimate). Taken averaged measurements will minimize errors from noise of the camera detector and other electrical components.

Reproducibility: Reproducibility is a function of both instrument repeatability, short and long term and repeatable procedures. Established exact procedures are easy to implement. Vibration isolation, especially <30 Hz vibrations which are difficult to remove, and reduction of acoustic noise in the < 100 Hz regime is also required.

Air-Turbulence and Thermal Gradients: These two affect reproducibility, yet are best treated separately since improving one can degrade the other. Thermal gradients are best removed by mixing the air, which causes turbulence and visa versa. Add to this the requirement to thermally soak the test part (see below) and a shielded cavity to reduce air-turbulence can set up thermal gradients. It is suggested the shield be constructed of soft materials to minimize transmitting acoustic noise into the cavity and live with the thermal gradients that will look like wedge in the part, and maybe a non-linear wedge top to bottom. If thermal gradient induced wedge is observed rotating the part will allow removal of this error.

Part Uncertainty

Germanium is a difficult material to measure for homogeneity due to its dn/dT and birefringence properties.

dn/dT

The sensitivity of index of refraction with temperature is critical since it directly affects the parameter measured in homogeneity-n. To achieve the Part Uc ~ 1 ppm, the temperature throughout the part must be in thermal equilibrium to <0.0014° K, very tight temperature control. Considering Ge has 30% the thermal conductivity of Aluminum and 6000% the thermal conductivity of Glass its ability to reach thermal equilibrium is superior to glass. Experience indicates that 100 mm diameter X 25 mm thick plate of glass takes 2 to 3 hours to reach thermal equilibrium. A Ge piece of the same size will reach equilibrium in <5 mins. Therefore after a 5 minute “soak” is applied, the main driver in dn/dT is the room temperature variations and gradients. The best test set up is devoid of drafts and temperature swings > 0.001° K/15 minute period. At these levels the test part is sensitive to radiative heating, including the 3.39um laser illumination!

Practically, thermal equilibrium in the test part must be maximized to improve the homogeneity test Uc. Once thermal equilibrium has been minimized to achieve < 5 ppm in the Test Part Uc need the operator focus on improvement in the Interferometer and Practice Uc.

[23] SCHOTT Technical Information Bulletin, TIE-26: Homogeneity of Glass, July 2004, page 2

Birefringence

Intrinsic birefringence is interpreted as polarization sensitive inhomogeneity. Intrinsic birefringence due to the manufacturing process of λ/2 was observed in carefully grown Ge crystals[24]. Birefringence is a critical element of homogeneity testing of Ge. See Appendix 2 for images of large scale intrinsic birefringence.

Thermal or mechanical stress birefringence can induce errors in the homogeneity measurements. Since intrinsic birefringence can be large we have great concern about stress induced birefringent errors. This is true for all material with a special caution noted for Ge. Therefore care must be taken mounting the part under test to assure that stress is not degrading the results. It is recommended that the test part be mounted so as to distribute mounting forces over a large area and minimize their magnitude.

Summary

A 3.39 um Twyman-Green interferometer can be expected to measure Germanium to $< \pm 5$ ppm 2σ with nominal care. If great care is taken and environment well controlled $< \pm 2$ ppm 2σ is possible. The major limitation to low ppm Uc results is the Ge material itself with particular caution to obtain thermal equilibrium to < 0.0014°. Awareness of the presence of intrinsic birefringence and the effect of polarization must be accounted for, and care taken to mount the test part to minimize stress induced birefringence.

Final Note

Establishing a baseline Uc for any set up can be achieved by running a long term repeatability test while monitoring temperature. This will bracket the Uc due to temperature variations, which are by far the largest contributor. It will also indicate if the part related Uc are under control. A long term repeatability test does not estimate the bias errors or practice errors, just the thermal variation errors.

[24] B. Depuydt, P. Boone, P. Union, P. Muys, D. Vyncke, & C. Goessens, “Interferometrical Characterization of Stress Birefringence in Germanium”, SPIE Vol. 3098 0277-786X1971, p 599 to 565

Appendix 1

Interferometric measurement of material homogeneity

Measurement setup

Interferometric measurement of material homogeneity is usually performed using a Fizeau or Twyman-Green interferometer. For simplicity and with no loss of generality a Fizeau interferometer will used in this analysis. A Fizeau has two flat references: Transmission Flat (TF) and Reference Flat (RF). A sample of material is placed inside the Fizeau cavity and series of measurements are taken. Sample of the tested material should be prepared in a form of a block with two flat-polished surfaces – the entrance surface (S1) and the exit surface (S2). Typically surfaces S1 and S2 should be polished to a surface figure better then l/2 (PV) with a slight wedge between them (see Table 1) in order to allow independent measurements of both surfaces.

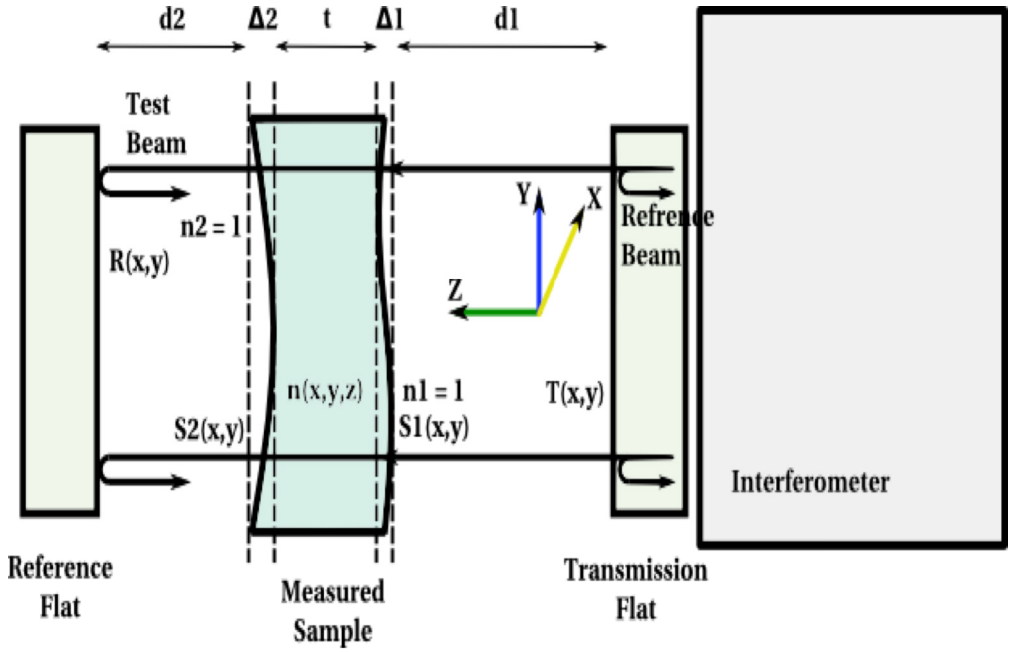

Figure 2. Diagram of he measurement setup.

With two flat surfaces of the sample it is possible to produce 6 measurements[25] combining pairs of reflections from the different surfaces. At least 3 measurements are needed to calculate homogeneity map of the measured sample.

For the PHOM measurement we are assuming location of the coordinate system as illustrated in the Figure 2. Functions T(x,y) and R(x,y) describe surface topographies (deviation from flatness) of the transmission and the reference flats, respectively. Analogously, S1(x,y) and S2(x,y) describe surface topography of the entrance and exit walls of the measured sample. Lets analyze a particular measurement that uses reflections from the transmission flat and the reference flat with the sample of thickness t placed inside the Fizeau cavity. The thickness of the sample block t could be defined as a distance between two surfaces perpendicular to the z-axis and delimiting maximum length of the sample completely filled with the material of the sample (position of the planes is marked by the two inner dashed lines in Figure 2). The outer dashed lines define the outer extend of the surfaces S1 and S2 along the z-axis – the distances $\Delta 1$ and $\Delta 2$ correspond to PV parameter of the surfaces profiles S1(x,y) and S2(x,y). The Optical Path Difference (OPD) measured by the interferometer can be defined as:

[25] See for example: Klaus Mantel, Johannes Schwider, “Interferometric Homogeneity Test Using Adaptive Frequency Comb Illumination”, DgaO Proceedings 2012 - http://www.dgao-proceedings.de - ISSN:1614-8436

$$OPD(x,y) = 2\int_0^{L(x,y)} n(x,y,z)\,dz$$

where L(x,y) is the length of the Fizeau cavity (distance between the transmission flat and reference flat) along the z axes[26].

We will assume the refractive index of the air $n_0(x,y) = 1$. Using the geometry of the setup shown in the Figure 1 we can calculate the OPD(x,y) as follow[27]:

$$\frac{OPD(x,y)}{2} = -T(x,y)\cdot n_0 + d1\cdot n_0 + S1(x,y)\cdot n_0 + \int_0^{\Delta 1 - S1(x,y)} n(x,y,z)\,dz + \int_0^{t} n(x,y,z)\,dz + \int_0^{S2(x,y)} n(x,y,z)\,dz + (\Delta 2 - S2(x,y))\cdot n_0 + d2\cdot n_0 + R(x,y)\cdot n_0$$

$$= -T(x,y) + S1(x,y) - S1(x,y)\cdot \bar{n}(x,y) + (t+\Delta 1)\cdot \bar{n}(x,y) + S2(x,y)\cdot \bar{n}(x,y) - S2(x,y) + R(x,y)$$

where the constant terms were dropped. Also, the distribution of the refractive index along the z-axis was replaced by it's "averaged" value $\bar{n}(x,y)$ such that $\bar{n}(x,y)\cdot t = \int_0^t n(x,y,z)\,dz$ i.e. we are assuming that the refractive index is constant along the z-axis at every point (x,y) of the sample.
The PHOM test is designed for homogeneity measurements of samples that are made of generally uniform material. In this case it is possible to define the variations of refractive index in the material as local departures from a nominal value n, constant for the entire sample $\bar{n}(x,y) = n + \overline{\Delta n}(x.y)$ where $\overline{\Delta n}(x.y) \ll n$. With this and with the assumption that the values of S1(x,y) and S2(x,y) are much smaller that thickness of the sample t, we can re-write the expression for the OPD as

$$\frac{OPD(x,y)}{2} = -T(x,y) + S1(x,y) - S1(x,y)\cdot n + (t+\Delta 1)\cdot \overline{\Delta n}(x,y) + S2(x,y)\cdot n - S2(x,y) + R(x,y)$$

$$= R(x,y) + S2(x,y)\cdot(n-1) + (t+\Delta 1)\cdot \overline{\Delta n}(x,y) - S1(x,y)\cdot(n-1) - T(x,y)$$

where the constant factors were dropped as well as the small contributions of terms $S1(x,y)\cdot\overline{\Delta n}(x,y)$ and $S2(x,y)\cdot\overline{\Delta n}(x,y)$ were neglected. This last assumption can be interpreted as adopting the assumption that the entire sample has a uniform thickness of $t+\Delta 1$. It should be noted that the thickness of the sample can not be established as a part of PHOM test by interferometric means and it has to be measured by a different method. Also, the wedge between the entrance and exit walls of the sample will contribute to the uncertainty of the sample thickness. We should conclude that the thickness t as it is defined in Figure 2 can not be really measured and thus we have to assume the thickness of the sample as $t_0 = t + \Delta 1 + \overline{t_w}$ where $\overline{t_w}$ is the thickness of the wedge averaged over the sample diameter.
Measurements that involve reflections from other pairs of surfaces can be treated in a similar way. By combining series of measurements it is possible to eliminate influence of S1 and S2 surfaces and the

[26] In order to get correct relationship between value of refractive index and the measured OPD, sign of the OPD map must be properly chosen – i.e. larger values of measured phase must correspond to larger values of the OPD. It is common practice in interferometry to invert this sign in order to describe topography of the measured surface in an inverted coordinate system.

[27] Similar approach can be found in Bob Oreb, Achim Leistner, GariLynn Billingsley, Bill Kells and Jordan Camp, "Interferometric measurement of refractive index inhomogeneity on polished sapphire substrates: application to LIGO-II", *Proc. SPIE* 4451, Optical Manufacturing and Testing IV, 414 (December 27, 2001);

geometry of the transmission and reference flats and calculate a two-dimensional map of distribution of the refractive index.

Appendix 2

Example of Intrinsic Birefringence

This demonstrates intrinsic birefringence within a crystal Ge plate[28]

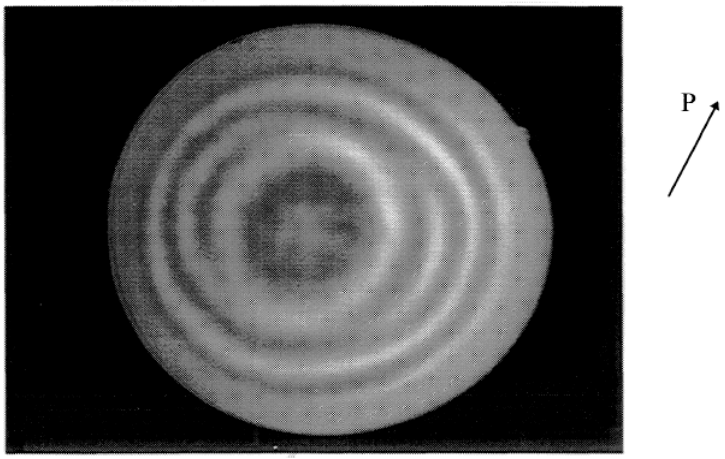

Figure 4: Interferogram of the transmitted wavefront for a disk with typical cylinder-symmetrical stress distribution. The incident polarization direction is indicated.

Notice how the fringes distort and disappear along the polarization axis noted by the arrow labeled P. This indicates inhomogeneity of nearly 1/4λ or a 8,500 nm error in this 45 mm thick disk. Therefore polarization must be carefully understood while assessing inhomogeneity in Ge.

[28]B. Depuydt, P. Boone, P. Union, P. Muys, D. Vyncke, & C. Goessens, “Interferometrical Characterization of Stress Birefringence in Germanium”, SPIE Vol. 3098 0277-786X1971, p 599 to 565